On a Characteristic of Data Transmission in Bus Network under Priority Processing with Waiting High Priority Data

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Abstract—This paper presents a characteristic of data transmission in bus network under a new priority processing. This priority processing is that a sending station waits for a high priority datum on waiting probability when there is no high priority data, and then sends the arrival high priority datum. In this bus network, each station is connected to a bus through a switching-hub which establishes a link from a sending station to a receiving station. When the receiving station is sending or receiving a datum, the link cannot be established, and then the sending station has to wait a little waiting time (stand-by time). Let me define the link probability as a constant and the waiting probability as a constant.

Taking note of one of the sending stations with two sending buffers for high priority data and low priority data, a balance equation of the probabilities of the number of waiting data at the ready time of sending is expressed in steady state. From the balance equation, for each priority level, probability of the number of waiting data, mean of the number of waiting data and mean waiting time are derived. The calculated values of mean waiting time are illustrated and they are compared with mean waiting time with simple priority processing. As a result, the influence of the waiting probability and the link probability on characteristic of data transmission is clarified.

Index Terms—Bus network, probability of the number of waiting data, mean of the number of waiting data, mean waiting time, priority processing, link probability, waiting probability.

I. INTRODUCTION

Bus network is a primary topology for local area network. Recently, in bus network, stations are connected to a bus through a switching hub which establishes a link from a sending station to a receiving station. In this network, a sending station has to wait for starting to send a datum when the transmission channel is occupied as in the case that the receiving station is sending or receiving a datum or other transmission occupies part of the transmission channel. For analysis of this bus network, I defined a constant link probability that a switching hub establishes a transmission channel, and then I reported a characteristic of data transmission [1], [2].

In the case of transmission of different kinds of data, such as text, sounds and graphics, priority processing is necessary. For priority processing, the proposal of MAC protocol with priority adjustment [3] and the simulation of MIN with priority routing [4] were studied. And, CAN with priority [5] and ad hoc network with priority [6] were reported. I also reported a characteristic of bus network with two priority levels [7]. Under this priority processing, high priority datum

is sent if it exists in the sending buffer, and low priority datum is sent if it exists in the sending buffer in the case that there is no high priority data. Let me call this processing a simple priority processing.

In this paper, I consider a new priority processing which makes mean waiting time of high priority data shorter. This priority processing is that the sending station waits for a high priority datum to arrive on constant waiting probability when there is no high priority data, and then sends the arrival high priority datum. Taking note of one of the sending stations which has two sending buffers for high priority data and low priority data, a balance equation of the probability of the number of waiting data is expressed. From the balance equation, for each priority level, probability of the number of waiting data, mean of the number of waiting data and mean waiting time from the arrival time at the sending buffer to the beginning time of sending are derived. Furthermore, the calculated values of mean waiting time are illustrated and mean waiting times under this priority processing are compared with those under the simple priority processing. As a result, the influence of the waiting probability and the link probability on characteristic of data transmission is clarified.

II. MODELING OF BUS NETWORK

The bus network shown in Fig. 1 is analyzed.

Fig. 1. Bus network.

In the network, each station has two infinite buffers for high and low priority data. The buffers are called sending buffers. Each priority data arrive at each sending buffer with Poisson process. The arrival ratios for high and low priority data are λ_1 and λ_2 , respectively. At the ready time of sending, if high priority data exist in the sending buffer and a link to a receiving station is established, the station sends the first high priority datum in the sending buffer. If not established, the station has a little waiting time which is called stand-by time. If there is no high priority data, the

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station waits a high priority datum on probability d and sends the arrival datum. On probability $(1-d)$, the station does not wait for a high priority datum and checks low priority data. If low priority data exist and the link is established, the station sends the first low priority datum in the sending buffer. If not established, the station has stand-by time. If there is no low priority data, the station has stand-by time. Defining the link probability as a constant q , the flowchart of data transmission is shown in Fig. 2.

Fig. 2. Flowchart of data transmission.

Taking note of one of the sending stations, the sending station is modeled as Fig. 3.

Fig. 3. Model of a sending station.

Transmission time is arbitrary distributed and the distribution function (DF) of the transmission time for high and low priority data are denoted by $H_1(t)$ and $H_2(t)$, respectively. The stand-by time is arbitrary distributed and the DF of the stand-by time is denoted by $H_c(t)$.

III. PROBABILITY OF THE NUMBER OF WAITING DATA

A. Balance Equation

Let P_{n_1, n_2} be the probability that the number of high priority waiting data is n_1 and the number of low priority waiting data is n_2 in each sending buffer at the ready time of sending. In steady state, the following balance equation is obtained.

$$\begin{aligned} P_{n_1, n_2} = & q \int \sum_{n_1'=1}^{\infty} \alpha_1(n_1' - n_1 + 1, t) \sum_{n_2'=0}^{n_2'+1} \alpha_2(n_2' - n_2, t) P_{n_1, n_2} dH_1(t) \\ & + (1-q) \int \sum_{n_1'=1}^{\infty} \alpha_1(n_1' - n_1, t) \sum_{n_2'=0}^{n_2'+1} \alpha_2(n_2' - n_2, t) P_{n_1, n_2} dH_c(t) \\ & + dq \int \alpha_1(n_1', t) \sum_{n_2'=0}^{n_2'+1} \alpha_2(n_2' - n_2, t) \left\{ \sum_{n_2=0}^{\infty} \alpha_2(m_2 - n_2, t) P_{0, n_2} dA(t) \right\} dH_1(t) \end{aligned}$$

$$\begin{aligned} & + d(1-q) \int \alpha_1(n_1' - 1, t) \sum_{n_2'=0}^{n_2'+1} \alpha_2(n_2' - n_2, t) \\ & \times \left\{ \sum_{n_2=0}^{\infty} \alpha_2(m_2 - n_2, t) P_{0, n_2} dA(t) \right\} dH_c(t) \\ & + (1-d) q \int \alpha_1(n_1', t) \sum_{n_2'=1}^{n_2'+1} \alpha_2(n_2' - n_2 + 1, t) P_{0, n_2} dH_2(t) \\ & + (1-d) (1-q) \int \alpha_1(n_1', t) \sum_{n_2'=1}^{n_2'+1} \alpha_2(n_2' - n_2, t) P_{0, n_2} dH_c(t) \\ & + (1-d) \int \alpha_1(n_1', t) \alpha_2(n_2', t) P_{0, 0} dH_c(t) \end{aligned} \quad (1)$$

In equation (1), $\alpha_1(n, t)$ and $\alpha_2(n, t)$ are the probabilities that n high priority data and n low priority data arrive at each sending buffer within time t , respectively.

$$\alpha_i(n, t) = \frac{e^{-\lambda_i t} (\lambda_i t)^n}{n!} \quad (i=1, 2) \quad (2a)$$

And, $A(t)$ is the DF of waiting time for a high priority datum expressed by equation (2b).

$$A(t) = 1 - e^{-\lambda_1 t} \quad (2b)$$

Taking $\sum_{n_1'=0}^{\infty} \sum_{n_2'=0}^{\infty}$ after multiplying $x_1^{n_1} x_2^{n_2}$ in both sides of equation (1) and transforming into a generating function form, the following equation is obtained.

$$\begin{aligned} G_I(x_1, x_2) = & \frac{1}{1 - \frac{q}{x_1} H_1^*(z) - (1-q) H_c^*(z)} \\ & \times [G_2(x_2) \{dq A^*(z_2) H_1^*(z) + d(1-q) x_1 A^*(z_2) H_c^*(z) \\ & + (1-d) \frac{q}{x_2} H_2^*(z) + (1-d)(1-q) H_c^*(z) - 1\} \\ & + P_{0,0} \{dq A^*(z_2) H_1^*(z) + d(1-q) x_1 A^*(z_2) H_c^*(z) \\ & + (1-d) H_c^*(z) - 1\}] \end{aligned} \quad (3)$$

where $H_i^*(z)$ ($i=1, 2$), $H_c^*(z)$ and $A^*(z_2)$ are the Laplace-Stieltjes Transformation (LST) of $H_i(t)$ ($i=1, 2$), $H_c(t)$ and $A(t)$, respectively, and

$$z = \lambda_1(1-x_1) + \lambda_2(1-x_2).$$

The generating functions of the probabilities of the number of waiting data are defined as follows.

$$G_I(x_1, x_2) = \sum_{n_1=1}^{\infty} \sum_{n_2=0}^{\infty} P_{n_1, n_2} x_1^{n_1} x_2^{n_2} \quad (4a)$$

$$G_2(x_2) = \sum_{n_2=1}^{\infty} P_{0, n_2} x_2^{n_2} \quad (4b)$$

B. Probabilities

When $x_i = x_2 = 1$ in equation (3), the following equations are obtained.

$$\begin{aligned} & \{q - q\lambda_1 h_i - (1-q)\lambda_1 h_c\} G_i(1,1) - \{dq\lambda_1 h_i + d(1-q) \\ & + d(1-q)\lambda_1 h_c + (1-d)q\lambda_1 h_2 + (1-d)(1-q)\lambda_1 h_c\} G_2(1,1) \quad (5a) \\ & - \{dq\lambda_1 h_i + d(1-q) + d(1-q)\lambda_1 h_c + (1-d)\lambda_1 h_c\} P_{0,0} = 0 \end{aligned}$$

$$\begin{aligned} & \{-q\lambda_2 h_i - (1-q)\lambda_2 h_c\} G_i(1,1) - \{dq\lambda_2 a + dq\lambda_2 h_i \\ & + d(1-q)\lambda_2 a + d(1-q)\lambda_2 h_c - (1-d)q + (1-d)q\lambda_2 h_2 \quad (5b) \\ & + (1-d)(1-q)\lambda_2 h_c\} G_2(1) - \{dq\lambda_2 a + dq\lambda_2 h_i \\ & + d(1-q)\lambda_2 a + d(1-q)\lambda_2 h_c + (1-d)\lambda_2 h_c\} P_{0,0} = 0 \end{aligned}$$

In equation (5a) and (5b), h_i , h_c and a are means of $H_i(t)$, $H_c(t)$ and $A(t)$, respectively.

Because the sum of all probabilities is equal to 1, equation (6) is obtained.

$$G_i(1,1) + G_2(1) + P_{0,0} = 1 \quad (6)$$

From equation (5a), (5b) and (6), the following probabilities are derived.

$$G_i(1,1) = \frac{NG_i}{DG_i} \quad (7a)$$

$$\begin{aligned} NG_i &= \lambda_1 h_c + d[1 - \lambda_2 h_2 + \lambda_2 h_c + a\lambda_1 \lambda_2 h_2 \\ &- a\lambda_1 \lambda_2 h_c + q\{-1 + \lambda_1 h_i + \lambda_2 h_2 - (\lambda_1 + \lambda_2)h_c\}] \quad (7aa) \end{aligned}$$

$$\begin{aligned} DG_i &= q[1 - \lambda_1 h_i - \lambda_2 h_2 + (\lambda_1 + \lambda_2)h_c] + d(1 - \lambda_2 h_2 \\ &+ \lambda_2 h_c + a\lambda_1 \lambda_2 h_2 - a\lambda_1 \lambda_2 h_c \quad (7ab) \\ &+ q\{-1 + \lambda_1 h_i + \lambda_2 h_2 - (\lambda_1 + \lambda_2)h_c\}] \end{aligned}$$

$$G_2(1) = \frac{NG_2}{DG_2} \quad (7b)$$

$$\begin{aligned} NG_2 &= \lambda_2 [qh_c + d\{q(a + h_i - a\lambda_1 h_i) \\ &+ h_c(1 - 2q - a\lambda_1 + qa\lambda_1)\}] \quad (7ba) \end{aligned}$$

$$\begin{aligned} DG_2 &= (1-d)q[q\{1 - \lambda_1 h_i - \lambda_2 h_2 + (\lambda_1 + \lambda_2)h_c\} \\ &+ d[1 - \lambda_2 h_2 + \lambda_2 h_c + \lambda_1 \lambda_2 h_2 - a\lambda_1 \lambda_2 h_c \quad (7bb) \\ &+ q\{-1 + \lambda_1 h_i + \lambda_2 h_2 - (\lambda_1 + \lambda_2)h_c\}]] \end{aligned}$$

$$P_{0,0} = 1 - G_i(1,1) - G_2(1) \quad (7c)$$

$G_i(1,1)$ is the probability that the number of high priority waiting data is one or more at the ready time of sending. $G_2(1)$ is the probability that the number of high priority waiting data is 0 and the number of low priority waiting data

is one or more. $P_{0,0}$ is the probability that there is no waiting data.

In the case of $d=0$, these probabilities are equal to the probabilities in the simple priority processing [15] reported before.

IV. MEAN OF THE NUMBER OF WAITING DATA

A. High Priority Level

From equation (3), $\frac{d}{dx_i} G_i(x_i, 1)|_{x_i=1}$ which is mean of the number of high priority waiting data in the sending buffer can be derived.

$$G_i(x_i, 1) = \frac{B_i(x_i)}{A_i(x_i)} \quad (8)$$

$$A_i(x_i) = 1 - \frac{q}{x_i} H_i^*(z_i) - (1-q)H_c^*(z_i) \quad (9a)$$

$$\begin{aligned} B_i(x_i) &= G_2(1)\{dqH_1^*(z_i) + d(1-q)x_i H_c^*(z_i) \\ &+ (1-d)qH_2^*(z_i) + (1-d)(1-q)H_c^*(z_i) - 1\} \quad (9b) \\ &+ P_{0,0}\{dqH_1^*(z_i) + d(1-q)x_i H_c^*(z_i) \\ &+ (1-d)H_c^*(z_i) - 1\} \end{aligned}$$

where $z_i = \lambda_1(1 - x_i)$.

When $x_i = 1$ after differentiating equation (8), the following equation is obtained.

$$\frac{d}{dx_i} G_i(x_i, 1)|_{x_i=1} = \frac{B_i''(1)A_i'(1) - B_i'(1)A_i''(1)}{2\{A_i'(1)\}^2} \quad (10)$$

where

$$A_i'(1) = q - q\lambda_1 h_i - (1-q)\lambda_1 h_c \quad (11a)$$

$$\begin{aligned} B_i'(1) &= G_2(1,1)\{dq\lambda_1 h_i + d(1-q) + d(1-q)\lambda_1 h_c \\ &+ (1-d)q\lambda_1 h_2 + (1-d)(1-q)\lambda_1 h_c\} \quad (11b) \\ &+ P_{0,0}\{dq\lambda_1 h_i + d(1-q) + d(1-q)\lambda_1 h_c + (1-d)\lambda_1 h_c\} \end{aligned}$$

and

$$A_i''(1) = -2q + 2q\lambda_1 h_i - q\lambda_1^2 h_i^{(2)} - (1-q)\lambda_1^2 h_c^{(2)} \quad (12a)$$

$$\begin{aligned} B_i''(1) &= G_2(1)\{dq\lambda_1^2 h_i^{(2)} + 2d(1-q)\lambda_1 h_c + d(1-q)\lambda_1^2 h_c^{(2)} \\ &+ (1-d)q\lambda_1^2 h_2^{(2)} + (1-d)(1-q)\lambda_1^2 h_c^{(2)}\} \quad (12b) \\ &+ P_{0,0}\{dq\lambda_1^2 h_i^{(2)} + 2d(1-q)\lambda_1 h_c + d(1-q)\lambda_1^2 h_c^{(2)} \\ &+ (1-d)\lambda_1^2 h_c^{(2)}\} \end{aligned}$$

$h_i^{(2)}$ and $h_c^{(2)}$ are the second moments of the DFs of $H_i(t)$ and $H_c(t)$, respectively.

B. Low Priority Level

Using $x_1 = \delta(x_2)$ which satisfies that the denominator of equation (3) is equal to 0, the following equation is obtained.

$$G_2(x_2) = \frac{B_2(x_2)}{A_2(x_2)} \quad (13)$$

$$\begin{aligned} A_2(x_2) = & 1 - dqA^*(z_2)H_1^*(\tilde{z}) \\ & - d(1-q)\delta(x_2)A^*(z_2)H_c^*(\tilde{z}) \\ & - (1-d)\frac{q}{x_2}H_2^*(\tilde{z}) - (1-d)(1-q)H_c^*(\tilde{z}) \end{aligned} \quad (14a)$$

$$\begin{aligned} B_2(x_2) = & P_{0,0}\{dqA^*(z_2)H_1^*(\tilde{z}) \\ & + d(1-q)\delta(x_2)A^*(z_2)H_c^*(\tilde{z}) \\ & + (1-d)H_c^*(\tilde{z}) - 1\} \end{aligned} \quad (14b)$$

where $z_2 = \lambda_2(1-x_2)$, $\tilde{z} = \lambda_1\{1-\delta(x_2)\} + \lambda_2(1-x_2)$, and $\delta(x_2)$ satisfies the following equation.

$$1 - \frac{q}{\delta(x_2)}H_1^*(\tilde{z}) - (1-q)H_c^*(\tilde{z}) = 0 \quad (15)$$

From equation (13), $\frac{d}{dx_2}G_2(x_2)|_{x_2=1}$ which is mean of the number of low priority waiting data in the case that the number of high priority waiting data is 0 can be derived.

When $x_2=1$ after differentiating equation (13), the following equation is obtained.

$$\frac{d}{dx_2}G_2(x_2)|_{x_2=1} = \frac{B_2''(1)A_2'(1) - B_2'(1)A_2''(1)}{2\{A_2'(1)\}^2} \quad (16)$$

where

$$\begin{aligned} A_2'(1) = & -dq\lambda_2a - dq(-h_1)\tilde{z}'_0 - d(1-q)\delta'(1) - d(1-q)\lambda_2a \\ & - d(1-q)(-h_c)\tilde{z}'_0 + (1-d)q - (1-d)q(-h_2)\tilde{z}'_0 \\ & - (1-d)(1-q)(-h_c)\tilde{z}'_0 \end{aligned} \quad (17a)$$

$$\begin{aligned} B_2'(1) = & P_{0,0}\{dq\lambda_2a + dq(-h_1)\tilde{z}'_0 + d(1-q)\delta'(1) \\ & + d(1-q)\lambda_2a + d(1-q)(-h_c)\tilde{z}'_0 + (1-d)(-h_c)\tilde{z}'_0\} \end{aligned} \quad (17b)$$

$$\tilde{z}'_0 = \tilde{z}'|_{x_2=1}$$

$$\begin{aligned} A_2''(1) = & -dq\lambda_2^2a^{(2)} - 2dq\lambda_2a(-h_1)\tilde{z}'_0 - dqh_1^{(2)}(\tilde{z}'_0)^2 \\ & - dq(-h_1)\tilde{z}''_0 - d(1-q)\delta''(1) - d(1-q)\delta'(1)\lambda_2a \\ & - d(1-q)\delta'(1)(-h_c)\tilde{z}'_0 - d(1-q)\delta'(1)\lambda_2a - d(1-q)\lambda_2^2a^{(2)} \\ & - d(1-q)\lambda_2a(-h_c)\tilde{z}'_0 - d(1-q)\delta'(1)(-h_c)\tilde{z}'_0 \\ & - d(1-q)\lambda_2a(-h_c)\tilde{z}'_0 - d(1-q)h_c^{(2)}(\tilde{z}'_0)^2 - d(1-q)(-h_c)\tilde{z}''_0 \\ & - 2(1-d)q + 2(1-d)q(-h_2)\tilde{z}'_0 - (1-d)qh_2^{(2)}(\tilde{z}'_0)^2 \\ & - (1-d)q(-h_2)\tilde{z}''_0 - (1-d)(1-q)h_c^{(2)}(\tilde{z}'_0)^2 \\ & - (1-d)(1-q)(-h_c)\tilde{z}''_0 \end{aligned} \quad (18a)$$

$$\begin{aligned} B_2''(1) = & P_{0,0}\{dq\lambda_2^2a^{(2)} + 2dq\lambda_2a(-h_1)\tilde{z}'_0 + dqh_1^{(2)}(\tilde{z}'_0)^2 + dq(-h_1)\tilde{z}''_0 \\ & + d(1-q)\delta''(1) + d(1-q)\delta'(1)\lambda_2a + d(1-q)\delta'(1)(-h_c)\tilde{z}'_0 \\ & + d(1-q)\delta'(1)\lambda_2a + d(1-q)\lambda_2^2a^{(2)} + d(1-q)\lambda_2a(-h_c)\tilde{z}'_0 \\ & + d(1-q)\delta'(1)(-h_c)\tilde{z}'_0 + d(1-q)\lambda_2a(-h_c)\tilde{z}'_0 + d(1-q)h_c^{(2)}(\tilde{z}'_0)^2 \\ & + d(1-q)(-h_c)\tilde{z}''_0 + (1-d)h_c^{(2)}(\tilde{z}'_0)^2 + (1-d)(-h_c)\tilde{z}''_0\} \end{aligned} \quad (18b)$$

$\tilde{z}''_0 = \tilde{z}''|_{x_2=1}$ and $a^{(2)}$ is the second moment of the DF of $A(t)$.

On the other hand, from $\tilde{z} = \lambda_1\{1-\delta(x_2)\} + \lambda_2(1-x_2)$ and equation (15) under $x_2=1$, the following equations are obtained.

$$\delta'_0 = \frac{\lambda_2(h_c + qh_1 - qh_c)}{q - q\lambda_1h_1 - (1-q)\lambda_1h_c} \quad (19a)$$

$$\tilde{z}'_0 = -\lambda_1\delta'_0 - \lambda_2 \quad (19b)$$

$$\delta''_0 = \frac{\tilde{z}'_0[-2(1-q)\delta'_0h_c + \tilde{z}'_0\{(1-q)h_c^{(2)} + qh_1^{(2)}\}]}{q - q\lambda_1h_1 - (1-q)\lambda_1h_c} \quad (19c)$$

$$\tilde{z}''_0 = -\lambda_1\delta''_0 \quad (19d)$$

From these equations, $\tilde{z}'_0$ and $\tilde{z}''_0$ can be calculated.

V. MEAN WAITING TIME

A. High Priority Level

The probability that the number of high priority waiting data at the end of sending data is n_1' is denoted by $R_{n_1'}$, and then the following equation is obtained.

$$\begin{aligned} R_{n_1'} = & \frac{1}{q[G_I(1,1) + d\{P_{0,0} + G_2(1)\}]} \\ & \times \{q \int_0^{\infty} \alpha_1(n_1' - n_1 + 1, t) \sum_{n_2=0}^{\infty} P_{n_1, n_2} dH_I(t) \\ & + dq \int_0^{\infty} \alpha_1(n_1', t) \sum_{n_2=0}^{\infty} P_{0, n_2} dH_I(t)\} \end{aligned} \quad (20)$$

Taking $\sum_{n_1'=0}^{\infty}$ after multiplying $x_1^{n_1'}$ in both sides in equation (20), and transforming into equation of the GF, equation (21) is obtained.

$$G_{R_I}(x_1) = \frac{1}{q[G_I(1,1) + d\{P_{0,0} + G_2(1)\}]} \times \left[\frac{1}{x_1} G_I(x_1,1) H_I^*(z_I) + d\{P_{0,0} + G_2(1)\} H_I^*(z_I) \right] \quad (21)$$

where $G_{R_I}(x_1) = \sum_{n_1'=0}^{\infty} R_{n_1'} x_1^{n_1'}$.

On the other hand, the probability that the number of high priority arrival data during waiting time and sending time for high priority data is n_1' is equal to $R_{n_1'}$, so that the following equation is obtained.

$$R_{n_1'} = \int_0^{\infty} \alpha_1(n_1', t) dW_I(t) \otimes H_I(t) \quad (22)$$

where $W_I(t)$ is the DF of waiting time for high priority data and $\otimes$ is a convolution operator.

Taking $\sum_{n_1'=0}^{\infty}$ after multiplying $x_1^{n_1'}$ in both sides of equation (22) and transforming into equation of the GF, equation (23) is obtained.

$$G_{R_I}(x_1) = W_I^*(z_I) H_I^*(z_I) \quad (23)$$

where $W_I^*(z_I)$ is the LST of $W_I(t)$.

From equation (21) and (23), $W_I^*(z)$ is obtained as follows.

$$W_I^*(z_I) = \frac{1}{G_I(1,1) + d\{P_{0,0} + G_2(1)\}} \times \left[\frac{1}{x_1} G_I(x_1,1) + d\{P_{0,0} + G_2(1)\} \right] \quad (24)$$

When $z_I = 0$ after differentiating equation (24) by z_I , the left side of equation is obtained as follows.

$$\frac{dW_I^*(z_I)}{dz_I} \Big|_{z_I=0} = -w_I \quad (25a)$$

where w_I is mean of $W_I(t)$.

On the other hand, the right side is obtained as follows.

$$-\frac{1}{\lambda_1} \frac{1}{G_I(1,1) + d\{P_{0,0} + G_2(1)\}} \times \left\{ -G_I(1,1) + \frac{d}{dx_I} G_I(x_I,1) \Big|_{x_I=1} \right\} \quad (25b)$$

From equation (25a) and (25b), mean waiting time for high priority data w_I is obtained.

$$w_I = \frac{1}{\lambda_1} \frac{1}{G_I(1,1) + d\{P_{0,0} + G_2(1)\}} \times \left\{ -G_I(1,1) + \frac{d}{dx_I} G_I(x_I,1) \Big|_{x_I=1} \right\} \quad (26)$$

where $G_I(1,1)$ and $\frac{d}{dx_I} G_I(x_I,1) \Big|_{x_I=1}$ have been already derived, so that mean waiting time w_I can be calculated.

B. Low Priority Level

Similar to high priority level, mean waiting time for low priority data is obtained as follows.

$$w_2 = \frac{1}{\lambda_2 G_2(1)} \left\{ \frac{d}{dx_2} G_2(x_2) \Big|_{x_2=1} - G_2(1) \right\} \quad (27)$$

$G_2(1)$ and $\frac{d}{dx_2} G_2(x_2) \Big|_{x_2=1}$ have been already derived,

so that mean waiting time w_2 can be calculated.

C. Numerical Examples

Fig. 4 shows mean waiting times w_I and w_2 for arrival ratio λ_1 in the case that $\lambda_1 = \lambda_2$, link probability $q=0.9$, waiting probability $d=0$, $h_I = h_2 = 5$, $h_c = 1$, and $H_i(t)$ and $H_c(t)$ obey uniform distributions.

Fig. 4. Mean waiting time w_I and w_2 ($q=0.9, d=0, h_I = h_2 = 5, h_c = 1, \lambda_1 = \lambda_2$, uniform distribution).

In Fig. 4, the lower line is mean waiting time w_I and the upper line is w_2 .

In the case that $H_i(t)$ and $H_c(t)$ obey exponential distributions, mean waiting times are shown in Fig. 5.

Fig. 5. Mean waiting time w_1 and w_2 ($q=0.9, d=0, h_1=h_2=5, h_c=1, \lambda_1 = \lambda_2$, exponential distribution).

Because waiting probability d is equal to 0, these figures are equal to those in simple priority processing [15].

Next, mean waiting times in the case of $\lambda_1 = \lambda_2, q=0.9, d=0.1, h_1 = h_2 = 5, h_c = 1$ and $H_i(t)$ and $H_c(t)$ obey uniform distributions is shown in Fig. 6.

Fig. 6. Mean waiting time w_1 and w_2 ($q=0.9, d=0.1, h_1=h_2=5, h_c=1, \lambda_1 = \lambda_2$, uniform distribution).

In Fig. 6, solid lines are mean waiting times in the case of $d=0.1$ and dashed lines are those in the case of $d=0$. By waiting high priority datum on probability $d=0.1$, mean waiting time w_1 becomes slightly shorter than that in the case of $d=0$, but mean waiting time w_2 becomes longer.

In order to make the influence on mean waiting time w_2 shorter, let me consider the case of $\lambda_2 = 0.1\lambda_1$. Fig. 7a shows mean waiting times in the case of $\lambda_2 = 0.1\lambda_1, q=0.9, d=0.1, h_1 = h_2 = 5, h_c = 1$ and $H_i(t)$ and $H_c(t)$ obey uniform distributions, and Fig. 7b shows mean waiting times in the case of $d=0.3$.

Both figures show similar to Fig. 6. In a comparison with Fig. 7a and 7b, mean waiting time w_1 in the case of $d=0.3$ becomes slightly shorter than that in the case of $d=0.1$. But, mean waiting time w_2 becomes longer.

Fig. 7a. Mean waiting time w_1 and w_2 ($q=0.9, d=0.1, h_1=h_2=5, h_c=1, \lambda_2 = 0.1\lambda_1$, uniform distribution).

Fig. 7b. Mean waiting time w_1 and w_2 ($q=0.9, d=0.3, h_1=h_2=5, h_c=1, \lambda_2 = 0.1\lambda_1$, uniform distribution).

VI. CONCLUSION

In this paper, a characteristic of data transmission in bus network under priority processing with waiting high priority datum is clarified. In the analysis, taking note of one of the sending stations, the waiting probability that the station waits for a high priority datum when there is no high priority waiting data in the ready time of sending is defined as a constant d , and the link probability that the link from the station to a receiving station is established is defined as a constant q .

In the analysis, at first, the balance equation of the probability of the number of waiting data is expressed. From the balance equation, for each priority level, the probabilities of the number of waiting data, the mean of the number of waiting data and the mean waiting time are derived. For the analysis of low priority, a root of the denominator of generating functions is used.

Some numerical examples of mean waiting time are illustrated, and then the influences of the waiting probability and the link probability on characteristics of data transmission are clarified. Furthermore, the effect of waiting probability is confirmed by comparing with mean waiting time with $d>0$ and that with $d=0$.

As the result, it is clarified that waiting probability makes mean waiting time of high priority data slightly shorter, but it makes mean waiting time of low priority data very long.

For further study, I consider other priority processing with waiting high priority data in which it is expected that mean waiting time of low priority data does not become very long.

For example, the following priority processing will be considered.

(1) Priority processing with waiting high priority data in which the sending station also sends a low priority datum after the station waits for a high priority datum and sends the datum.

(2) Priority processing with waiting high priority data in which the sending station waits for a high priority datum in the case that both priority data do not exist.

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