Normalized Least-Mean-Square Algorithm with a Pseudo-Fractional Number of Orthogonal Correction Factors

Sang Mok Jung, Ji-Hye Seo, and Poo Gyeon Park

Abstract—**This paper proposes a normalized LMS algorithm (NLMS) that automatically determines the number of orthogonal correction factors (OCFs) by using a pseudo-fractional method, which relaxes the constraint that the number of OCFs in the NLMS algorithm must be integral and introduces the concept of a pseudo-fractional OCF number in the adaptation rule. The pseudo-fractional OCF number is adjusted by using the difference between the averages of the accumulated squared-output errors. The experimental results show that the proposed algorithm has not only a fast convergence rate but also a small steady-state estimation error with low computational complexity in comparison to existing algorithms with multiple input vectors.**

Index Terms—**Adaptive filters, normalized LMS with orthogonal correction factors (NLMS-OCF), pseudo-fractional OCF number, pseudo-fractional method.**

I. INTRODUCTION

The normalized LMS (NLMS) algorithm is widely used in adaptive filtering owing to its low computational complexity and ease of implementation [1], [2]. However, its convergence is deteriorated by colored input signals. To overcome this disadvantage, several algorithms such as the affine projection algorithm (APA) [3], the partial rank algorithm (PRA) [4], and the normalized LMS algorithm with orthogonal correction factors (NLMS-OCF) [5] have been developed over the decades. These algorithms update the weights on the basis of multiple input vectors, while the NLMS algorithm updates the weights on the basis of a single input vector. They achieve fast convergence rate, but have high computational complexity and a large steady-state estimation error because of the multiple input vectors.

To solve these problems, several methods have been suggested that adjust the number of input vectors in the updating procedure [6]-[8]. The APA with dynamic selection of input vectors (DS-APA) [6] and the family of APAs with dynamic selection of input vectors (DS-N-APA) [8] suggest the ideal selection criterion of the input vectors and perform the updating procedure with the selected input vectors. These algorithms can achieve lower computational complexity and smaller steady-state estimation error than other existing algorithms using multiple input vectors; however, they still have a larger steady-state estimation error than the NLMS algorithm.

This paper proposes the NLMS-OCF algorithm that determines the number of orthogonal correction factors (OCFs) by employing a pseudo-fractional method that was motivated from the concept of the pseudo-fractional tap length [9]. The pseudo-fractional method relaxes the constraint on the conventional NLMS-OCF algorithm that the number of OCFs must be integral, and it determines the number of OCFs by using the difference between the averages of the accumulated squared-output errors. By using both integral and pseudo-fractional numbers of OCFs, the proposed algorithm ensures a fast convergence rate and a small steady-state estimation error. In addition, it has lower computational complexity than the existing algorithms with multiple input vectors.

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II. NORMALIZED LMS ALGORITHM WITH ORTHOGONAL CORRECTION FACTORS

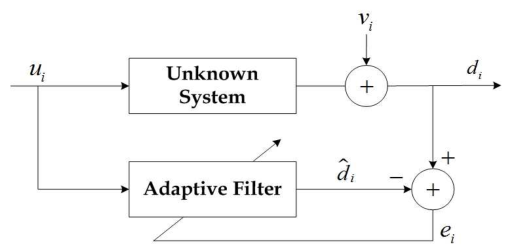

Fig. 1. Adaptive filtering structure.

Fig. 1 shows an adaptive filtering structure in the system identification application. The objective is to estimate an n-dimensional weight vector $\hat{\mathbf{w}}_i$ that makes the estimated error $e_i = d_i - \hat{d}_i$ as small as possible in the mean-squared-error sense. The estimated output and the true output can be expressed as $\hat{d}_i = \mathbf{u}_i^T \hat{\mathbf{w}}_i$ and $d_i = \mathbf{u}_i^T \mathbf{w} + v_i$, respectively, where $\mathbf{w}$ is an unknown column vector that we expect to estimate, $\mathbf{u}_i = [u_i, u_{i-1}, \ldots, u_{i-n+1}]^T$ denotes an n-dimensional input column vector at the i-th iteration, and v_i accounts for measurement noise with variance σ_v^2.

The weight updating equation of the NLMS-OCF algorithm is:

$$\hat{\mathbf{w}}_{i+1} = \hat{\mathbf{w}}_i + \bar{\mu}_0 \mathbf{u}_i + \bar{\mu}_1 \mathbf{u}_i^{(1)} + \cdots + \bar{\mu}_M \mathbf{u}_i^{(M)}, i \geq M < n \quad (1)$$

where $\mathbf{u}_i^{(j)}$ ($j = 1, 2, \ldots, M$) are the components of $\mathbf{u}_{i-jD}$ that are orthogonal to $\mathbf{u}_i, \mathbf{u}_{i-D}, \mathbf{u}_{i-2D}, \ldots, \mathbf{u}_{i-(j-1)D}$ and can be computed using the Gram-Schmidt procedure [10], M is the number of OCFs, and D is the delay between the input vectors used for the updating procedure. The value of D is chosen to be $\lceil M / 2 \rceil$ in this paper, where the operator $\lceil . \rceil$ rounds to the nearest integer towards infinity. Moreover, $\bar{\mu}_k$ ($k = 0, 1, \ldots, M$) is calculated according to

$$\bar{\mu}_k = \begin{cases} \dfrac{\mu e_i}{\mathbf{u}_i^T \mathbf{u}_i} & \text{for } k = 0, \text{if } \|\mathbf{u}_i\| \neq 0 \\ \dfrac{\mu e_i^{(k)}}{\mathbf{u}_i^{(k)T} \mathbf{u}_i^{(k)}} & \text{for } k = 1, 2, \ldots, M, \text{if } \|\mathbf{u}_i^{(k)}\| \neq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where $0 < \mu < 2$ is the step size,

$$e_i = d_i - \mathbf{u}_i^T \hat{\mathbf{w}}_i,$$
$$e_i^{(k)} = d_{i-kD} - \mathbf{u}_{i-kD}^T \hat{\mathbf{w}}_i^{(k)}, \text{and}$$
$$\hat{\mathbf{w}}_i^{(k)} = \hat{\mathbf{w}}_i + \bar{\mu}_0 \mathbf{u}_i + \bar{\mu}_1 \mathbf{u}_i^{(1)} + \cdots + \bar{\mu}_{k-1} \mathbf{u}_i^{(k-1)}$$

III. Proposed Algorithm: Normalized LMS with Pseudo-Fractional Number of Orthogonal Correction Factors

In this section, we introduce the concept of a pseudo -fractional number to control the number of OCFs, M, in the NLMS-OCF algorithm and we propose a pseudo-fractional method based on this concept. Moreover, the entire procedure of the proposed algorithm is summarized.

A. Pseudo-Fractional Method

The performance of the NLMS-OCF algorithm is affected by the number of orthogonal correction factors. The NLMS-OCF algorithm with a large M has fast convergence rate in the transient state but a large estimation error in the steady state. Conversely, the NLMS-OCF algorithm with a small M converges slowly but has a small steady-state estimation error. Therefore, determining the proper value of M is a very important issue when using the NLMS-OCF algorithm.

Generally, the NLMS-OCF algorithm has a constraint that the value of M must be integral. Because of this constraint on M, we cannot apply a small leaky factor in the adaptation rule for M. If the possible values of M can be considered to include not only integers but also non-integers, we can apply the leaky factor to the adaptation rule for M. Based on such motivation, this paper proposes a pseudo-fractional method that introduces the concept of pseudo-fractional OCF numbers to determine M by considering its properties.

The pseudo-fractional method introduces both the integral OCF number and the pseudo-fractional OCF number. The integral OCF number is needed to perform the updating procedure in the proposed algorithm, and the pseudo-fractional OCF number is needed to obtain the integral OCF number. The integral OCF number remains unchanged until the change of the pseudo-fractional OCF number accumulates to some extent. The pseudo-fractional OCF number is obtained by comparing the averages of the accumulated squared-errors and by using the leaky factor. When the difference between the integral and pseudo-fractional OCF numbers becomes greater than a predetermined value, the integral OCF number can be obtained from the integral part of the pseudo-fractional OCF number. The proposed method dynamically adjusts M and leads to the improvement of the performance in terms of the convergence rate and the steady-state estimation error.

Specifically, the pseudo-fractional OCF number at iteration i is denoted by $M_{f,i}$ and is computed with the following adaptation rule:

$$M_{f,i} = \begin{cases} (M_{f,i-1} - \alpha) - \\ \gamma[AASE_{M_i} - AASE_{M_i - \Delta}], & \text{if } M_i \geq \Delta \\ (M_{f,i-1} - \alpha) - \\ \gamma[AASE_{M_i + \Delta} - AASE_{M_i}], & \text{otherwise} \end{cases} \quad (3)$$

where α and γ are small positive numbers that satisfy $\alpha < \gamma$, and M_i is the integral OCF number at iteration i. The average of the accumulated squared-error (AASE) is defined as

$$AASE_{M_i} = \frac{\sum_{l=0}^{M_i} e_i^{(l)2}}{M_i + 1} \quad (4)$$

where $e_i^{(0)}$ represents e_i.

Furthermore, Δ is a positive integer that plays an important role in the pseudo-fractional method. A large value of Δ yields a fast convergence rate but a large steady-state estimation error. Conversely, a small Δ results in slow convergence rate but provides a small steady-state estimation error. In order to meet the conflicting requirements of fast convergence rate and small estimation error, the value of Δ is set to unity in this paper.

Then, the integral OCF number is determined according to

$$M_{i+1} = \begin{cases} \max(\min(\lfloor M_{f,i} \rfloor, M_{\max}), 0), & \text{if } \zeta \geq \delta \\ M_i, & \text{otherwise} \end{cases} \quad (5)$$

where the operator $\lfloor . \rfloor$ rounds the number to the nearest integer, δ is the threshold parameter that is set to unity in this paper, and $\zeta = | M_i - M_{f,i} |$. It should be noted that M_i is updated to satisfy, $0 \leq M_i \leq M_{\max}$ where $M_{\max}$ is the maximum number of OCFs.

B. Updating Procedure

From the method proposed above, the updating equation of the proposed algorithm becomes

$$\hat{\mathbf{w}}_{i+1} = \hat{\mathbf{w}}_i + \bar{\mu}_0 \mathbf{u}_i + \bar{\mu}_1 \mathbf{u}_i^{(1)} + \cdots + \bar{\mu}_{M_i} \mathbf{u}_i^{(M_i)} \quad (6)$$

where M_i is determined by the pseudo-fractional method. If M_i is zero, the proposed algorithm will operate as the NLMS algorithm.

The entire procedure of the proposed algorithm is described in Table I.

TABLE I: THE PROPOSED ALGORITHM

Initialization: Choose an arbitrary $\hat{\mathbf{w}}_0$. α, γ : user defined small positive numbers $M_0 = M_{f,0} = M_{\max}$ $(M_{\max} < n)$, and $\mu \in (0,2)$.
For each new input u_i **:** $e_i = d_i - \mathbf{u}_i^T \hat{\mathbf{w}}_i$ $\lVert\mathbf{u}_i\rVert^2 = \lVert\mathbf{u}_{i-1}\rVert^2 + u_i^2 - u_{i-n}^2$ $\bar{\mu}_0 = \frac{\mu e_i}{\lVert\mathbf{u}_i\rVert^2}$ $\hat{\mathbf{w}}_{i+1}^{(1)} = \hat{\mathbf{w}}_i + \bar{\mu}_0 \mathbf{u}_i$ $\mathbf{u}_i^{(0)} = \mathbf{u}_i$ **for** k=1,2,… , M_i , **do** $\mathbf{u}_i^{(k)} = \mathbf{u}_{i-kD} - \sum_{l=0}^{k-1} \frac{\mathbf{u}_{i-kD}^T \mathbf{u}_i^{(l)}}{\lVert\mathbf{u}_i^{(l)}\rVert^2} \mathbf{u}_i^{(l)}$ $e_i^{(k)} = d_{i-kD} - \mathbf{u}_{i-kD}^T \hat{\mathbf{w}}_{i+1}^{(k)}$ $\bar{\mu}_k = \begin{cases} \frac{\mu e_i^{(k)}}{\lVert\mathbf{u}_i^{(k)}\rVert^2}, & \text{if } \lVert\mathbf{u}_i^{(k)}\rVert \neq 0 \\ 0, & \text{otherwise} \end{cases}$ $\hat{\mathbf{w}}_{i+1}^{(k+1)} = \hat{\mathbf{w}}_{i+1}^{(k)} + \bar{\mu}_k \mathbf{u}_i^{(k)}$ **end** $\hat{\mathbf{w}}_{i+1} = \hat{\mathbf{w}}_{i+1}^{(M_i+1)}$ **if** $M_i \geq \Delta$ $M_{f,i} = (M_{f,i-1} - \alpha) - \gamma[AASE_{M_i} - AASE_{M_i-\Delta}]$ **else if** $M_{f,i} = (M_{f,i-1} - \alpha) - \gamma[AASE_{M_i+\Delta} - AASE_{M_i}]$ **end** **if** $M_i - M_{f,i} \geq \delta$ $M_{i+1} = M_i - 1$ **else if** $M_{f,i} - M_i \geq \delta$ $M_{i+1} = M_i + 1$ **end** M_{i+1} = max(min($M_{i+1}, M_{\max}$), 0) **end**

Remark: The proposed algorithm is designed for stationary environments. However, when the target system is changed, a re-initialization is required in order to achieve fast tracking performance. The proposed algorithm adopts the re-initialization method used in [11], but with modifications. The method of re-initialization is described in Table II.

TABLE II: RE-INITIALIZATION METHOD

$e_{th} = \mu\sigma_v^2 M_{\max} / (2-\mu)$, flag = 0, $e_{avg} = e_0^2$ $\lambda, \alpha_1, \alpha_2$: user defined
For each i **do** **if** $e_i^2 < \alpha_1 \times e_{th}$ flag =1 **else if** flag = 1 and $\alpha_2 \times e_{avg} < e_i^2$ flag =0, $e_{avg} = e_i^2$, $M_i = M_{\max}$, $M_{f,i} = M_{\max}$ **end** $e_{avg} \leftarrow \lambda e_{avg} + (1-\lambda)e_i^2$ **end**

IV. EXPERIMENTAL RESULT

To illustrate the performance of the proposed algorithm, we carried out computer simulations of a channel estimation. The unknown channels were randomly generated by a moving average model with 32 taps (n = 32). The adaptive filter and the unknown channel were assumed to have the same number of taps. The initial number of OCFs for the proposed algorithm, M_0, was set to 15, which is the maximum number of OCFs used in the conventional NLMS-OCF algorithm. The input signal u_i was generated by filtering a white, zero-mean, Gaussian random sequence through the following systems: $G_1(z) = 1/(1-0.9z^{-1})$, $G_2(z) = (1+0.6z^{-1})/(1+1.0z^{-1}+0.21z^{-2})$. The measurement noise υ_i was added to y_i with a signal-to-noise ratio (SNR) of 30dB, where the SNR is defined by $10\log_{10}(E[y_i^2]/E[\upsilon_i^2])$ and $y_i = \mathbf{u}_i^T \mathbf{w}$. Additionally, we assumed that the noise variance σ_υ^2 is known, because this can be estimated during silences in many practical applications [12]-[14]. The mean-squared deviation (MSD), $E\lVert\mathbf{w}-\hat{\mathbf{w}}_i\rVert^2$, was calculated to indicate the performance of the proposed algorithm and was obtained by ensemble averaging over 100 independent trials. The simulations were performed with $M_{\max} = 15$, $\mu = 0.5$, $D = 8$. Furthermore, to check the tracking performance of the proposed algorithm, we suddenly change the coefficients of the unknown filter taps from $\mathbf{w}$ to $-\mathbf{w}$ at time i=10000.

A. Performance Comparison

From Fig. 2 and Fig. 3, we confirm that the NLMS-OCF has a best performance when the number of OCF is decremented from a large number to a small number.

Fig. 4 and Fig. 5 show the MSD curves of the normalized LMS, the conventional APA, the conventional NLMS-OCF, the DS-APA, the DS-N-APA, and the proposed algorithm with the input signal generated by $G_1(z)$ and $G_2(z)$. As shown in the figures, the convergence rate of the proposed algorithm is almost the same as that of the APA, the NLMS-OCF, the DS-APA, and the DS-N-APA. However, the steady-state estimation errors of the NLMS-OCF, the DS-APA, and the DS-N-APA are large, because these algorithms use multiple input vectors even in the steady state. Conversely, the proposed algorithm has the smallest estimation error in the steady state as compared to other members of the APA family.

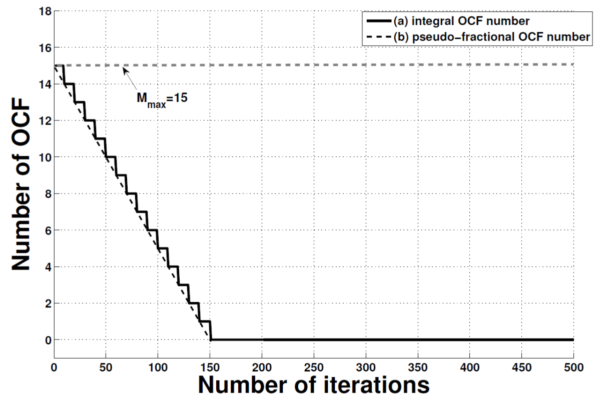

Fig. 2. The number of integral OCF and pseudo-fractional OCF for one trial in the proposed algorithm (The input signal is generated with $G_1(z)$, $n = 32$, SNR = 30dB).

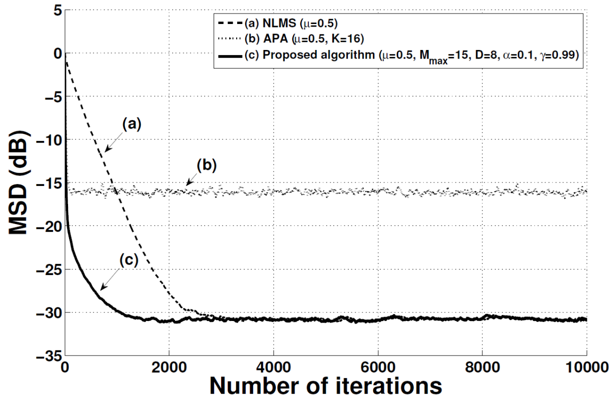

Fig. 3. The MSD curves of the NLMS, the APA, and the proposed algorithm (The input signal is generated with $G_1(z)$, $n = 32$, SNR = 30dB).

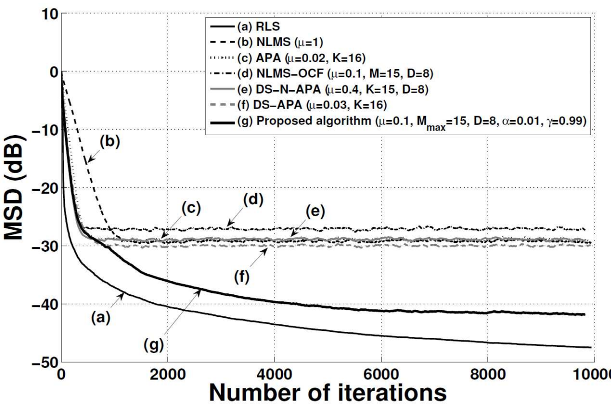

Fig. 4. The MSD curves of the RLS, the NLMS, the APA, the NLMS-OCF, the DS-APA, the DS-N-APA, and the proposed algorithm (The input signal is generated with $G_1(z)$, $n = 32$, SNR = 30dB).

Fig. 6 shows the tracking capability of the proposed algorithm when the unknown system is suddenly changed. As it is shown, we can confirm that the proposed algorithm keeps the performance for tracking the changed weight without degradation of the convergence rate or the steady-state estimation errors.

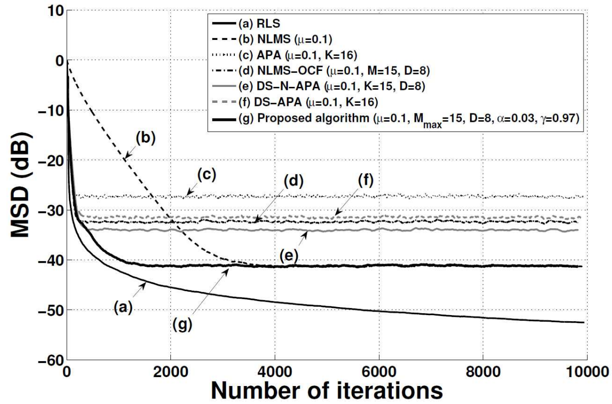

Fig. 5. The MSD curves of the RLS, the NLMS, the APA, the NLMS-OCF, the DS-APA, the DS-N-APA, and the proposed algorithm (The input signal is generated with $G_2(z)$, $n = 32$, SNR = 30dB).

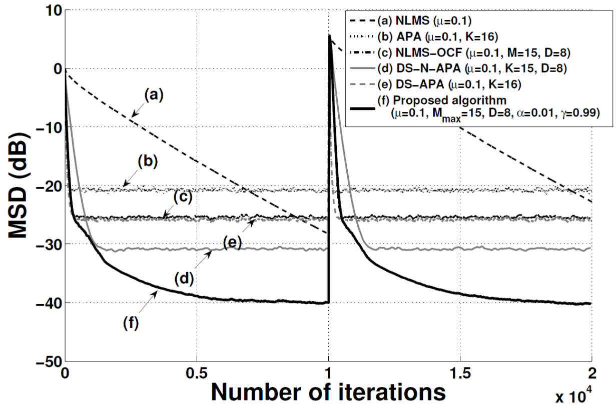

Fig. 6. The MSD curves of the NLMS, the APA, the NLMS-OCF, the DS-APA, the DS-N-APA, and the proposed algorithm (The input signal is generated with $G_1(z)$, $n = 32$, SNR = 30dB).

Fig. 7 shows the MSD curves of the normalized LMS, the conventional APA, the conventional NLMS-OCF, the DS-APA, the DS-N-APA, and the proposed algorithm with the input signal generated by $G_1(z)$ when the SNR is 10dB. As shown in the figure, the proposed algorithm has better performance than the existing APAs when the SNR is set lower.

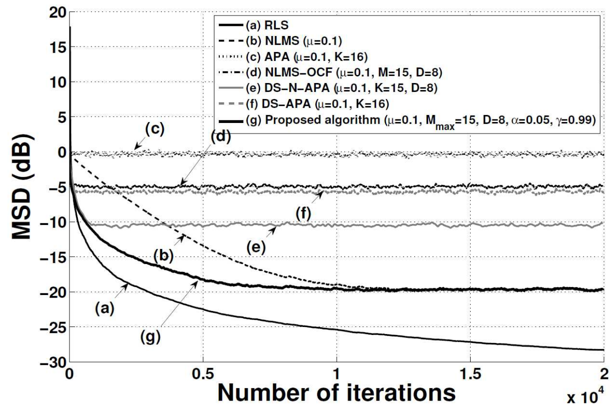

Fig. 7. The MSD curves of the RLS, the NLMS, the APA, the NLMS-OCF, the DS-N-APA, the DS-APA, and the proposed algorithm (The input signal is generated with $G_1(z)$, $n = 32$, SNR = 30dB).

B. Computational Complexity

Table III shows the computational complexity for each iteration of the conventional NLMS-OCF, the DS-N-APA, and the proposed algorithm. The numbers of OCFs for the NLMS-OCF algorithm, the DS-N-APA, and the proposed algorithm are M, M_k, and M_i, respectively.

TABLE III: COMPUTATIONAL COMPLEXITY OF THE NLMS-OCF, THE DS-N-APA, AND THE PROPOSED ALGORITHM

	Addition/Subtraction	Multiplication/Division
RLS [2]	n^2+3n	n^2+5n+2
NLMS [2]	$3n$	$3n+2$
APA [3]	$(K^2+2K)n+K^3$	$(K^2+2K)n+K^3+K^2$
NLMS-OCF [5]	$2n+2$ $+M(M+1)\times(2n-1)/2$ $+M(3n-1)$	$2n+4$ $+M(M+1)\times(2n+1)/2$ $+M(3n+2)$
DS-N-APA [8]	$(M+1)(n+1)+2n-1$ $+M_k(M_k+1)(2n-1)/2$ $+M_k(3n-1)$	$(M+1)n+2n+2$ $+M_k(M_k+1)(2n+1)/2$ $+M_k(3n+2)$
Proposed Algorithm	$2n+2$ $+M_i(M_i+1)\times(2n-1)/2$ $+M_i(3n-1)+M_i+5$	$2n+4$ $+M_i(M_i+1)\times(2n+1)/2$ $+M_i(3n+2)+M_i+4$

Fig. 8 shows the accumulated numbers of multiplications. The overall number of multiplications is much lower for the proposed algorithm than for the NLMS-OCF algorithm or the DS-N-APA, because the number of OCFs in the steady state is much smaller for the proposed algorithm than for the NLMS-OCF algorithm or the DS-N-APA.

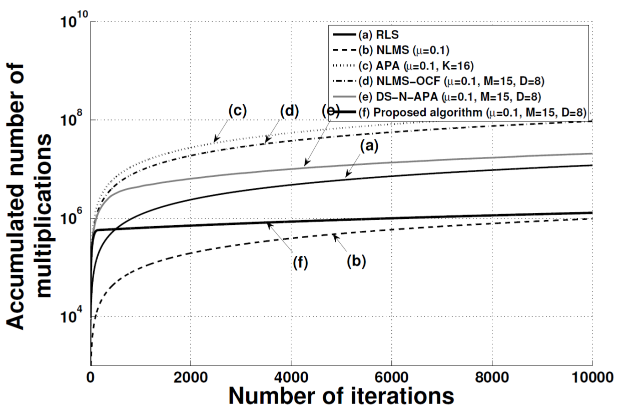

Fig. 8. Accumulated numbers of multiplications for the RLS, the NLMS, the NLMS-OCF, the DS-N-APA, and the proposed algorithm.

C. Acoustic Echo Cancellation

In order to verify the performance of the proposed algorithm for a non-stationary input signal, we also performed an experiment with an input signal of speech sampled at 8 kHz and an acoustic echo path of length n = 1024. The simulation was performed with $M_{\max}=7$, $\mu=1$, $D=4$, $\alpha=0.001$. Fig. 9 shows the exact impulse response of the room echo path that should be identified. Fig. 10 shows the MSD curves of the APA, the NLMS-OCF with a different M, and the proposed algorithm. As shown, we can confirm that the proposed algorithm performs well for a non-stationary signal such as speech.

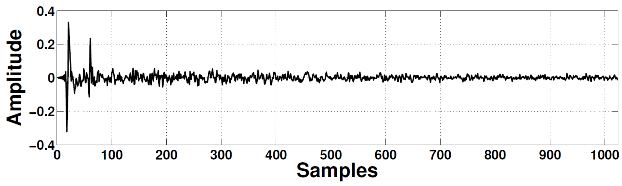

Fig. 9. Acoustic impulse response of a room used in the simulation.

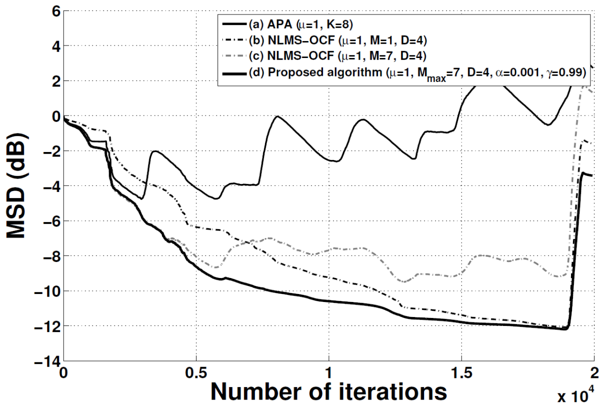

Fig. 10. MSD curves of the APA, the NLMS-OCF, and the proposed algorithm. (The input is a speech signal at 8 kHZ with n = 1024 and SNR = 30dB).

V. CONCLUSION

This paper has proposed an NLMS-OCF algorithm that adjusts the number of OCFs through a pseudo-fractional method. This method dynamically adjusts the number of OCFs using the proposed adaptation rule and relaxes the constraint that the number of OCFs must be integral. The proposed adaptation rule uses not only an integral OCF number but also a pseudo-fractional OCF number, and it determines the number of OCFs by using the difference between the averages of the accumulated squared-output errors. The proposed method leads to a fast convergence rate, a low steady-state estimation error, and low computational complexity for colored inputs. The experimental results show the improved performance of the proposed algorithm in comparison to the existing family of APAs.

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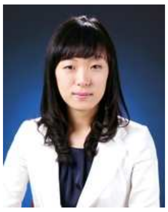

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